

Scale Transformation, Modified Gravity, and Brans-Dicke Theory

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Abstract A model of Einstein-Hilbert action subject to the scale transformation is studied. By introducing a dilaton field as a means of scale transformation a new action is obtained whose Einstein field equations are consistent with traceless matter with non-vanishing modified terms together with dynamical cosmological and gravitational coupling terms. The obtained modified Einstein equations are neither those in $f(R)$ metric formalism nor the ones in $f(\mathcal{R})$ Palatini formalism, whereas the modified source terms are *formally* equivalent to those of $f(\mathcal{R}) = \frac{1}{2}\mathcal{R}^2$ gravity in Palatini formalism. The correspondence between the present model, the modified gravity theory, and Brans-Dicke theory with $\omega = -\frac{3}{2}$ is explicitly shown, provided the dilaton field is condensed to its vacuum state.

Keywords Scale transformation · Modified gravity · Brans-Dicke theory

1 Introduction

Conformal invariance has played a key role in the study of G and Λ varying theories raised by Dirac [1–3]. Bekenstein was the first who introduced this possibility and tried to resolve G -varying problem [4]. The main point in this idea is the assumption that the time variation of couplings is generated by the dynamics of a cosmological scalar (dilaton) field. For example, Damour et al. have constructed a generalized Jordan-Brans-Dicke model in which the dilaton field couples with different strengths to the visible and dark matter [5]. On the other hand, Bertolami [6] has introduced a model in which both gravitational coupling and cosmological term are time dependent. Conformal invariance implies that the gravitational theory is invariant under local changes of units of length and time. These local transformations relate different unit systems or conformal frames via space time dependent conformal factors, and these unit systems are dynamically distinct. This usually leads to the variability of the fundamental constants. Recently, it is shown that one can use this dynamical distinction between two unit systems usually used in cosmology and particle physics to alleviate the cosmological constant and coincidence problems [7–10].

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Unlike the above models based on conformal invariance, the purpose of present paper is to study a gravitational model in which *scale transformations* play the key role in obtaining dynamical G and Λ . A scale transformation is different from a conformal transformation. A conformal transformation is viewed as “stretching” all lengths by a space time dependent conformal factor, namely a “unit” transformation. But, a scale transformation is just a rescaling of metric by a space time dependent conformal factor, and all lengths are assumed to remain unchanged. This kind of transformation is not a “unit” transformation; it is just a dynamical rescaling (enlargement or contraction) of a system. We will take a non-scale invariant gravitational action with a *cosmological constant* in which gravity couples minimally to a dimensionless dilaton field, and matter couples to a metric which is conformally related, through the dilaton field, to the gravitational metric. Then by a scale transformation, through the dilaton field, we obtain a new action in which gravity couples non-minimally to the dilaton field and matter couples to the gravitational metric. The Einstein equations reveal a cosmological term and a gravitational coupling which are dynamically dependent on the dilaton field (or conformal factor) satisfying the field equation with a Higgs type potential. The symmetry breaking in this potential may occur for positive cosmological constant and leads to the vacuum condensation of the dilaton field. By putting this vacuum state of the dilaton field in the Einstein-dilaton equations we obtain the modified Einstein equations in the spirit of current $f(R)$ theories of gravity [11–15]. Then, we study the characteristics of these equations and examine the correspondence with equations obtained in metric and Palatini formalisms in one hand, and Brans-Dicke theory on the other hand. It is very appealing to study the cosmology of this model and examine the correspondence with those recent interesting works on the cosmology of modified gravity theory and Brans-Dicke theory, for example the agegraphic dark energy density in the $f(R)$ gravity and holographic dark energy density in the Brans-Dicke theory [16–18].

2 Einstein-Hilbert Action and Scale Transformation

We start with the following action¹

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} [R - 2\bar{\Lambda} + \alpha g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma] d^4x + S_m(e^{2\sigma} g_{\mu\nu}), \tag{1}$$

where Einstein-Hilbert action with metric $g_{\mu\nu}$ is minimally coupled to a dimensionless dilaton field σ , and the matter is coupled to gravity with the metric $e^{2\sigma} g_{\mu\nu}$ which is conformally related to the metric $g_{\mu\nu}$ ². The parameters $\bar{\kappa}^2$ and $\bar{\Lambda}$ are gravitational coupling and cosmological constants, respectively, and α is to be determined later. Variation with respect to $g_{\mu\nu}$ and σ yields

$$G_{\mu\nu} + \bar{\Lambda} g_{\mu\nu} = \bar{\kappa}^2 \tilde{T}_{\mu\nu} + \tau_{\mu\nu}, \tag{2}$$

$$\square \sigma = \frac{\bar{\kappa}^2}{\alpha} \tilde{T}, \tag{3}$$

where

$$\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m(e^{2\sigma} g_{\mu\nu})}{\delta g^{\mu\nu}}, \tag{4}$$

¹We will use the sign convention $g_{\mu\nu} = \text{diag}(-, +, +, +)$ and units where $c = 1$.

²The idea of coupling matter to conformally related metrics has already been proposed by some authors [5].

$$\tau_{\mu\nu} = \alpha \left(\frac{1}{2} g_{\mu\nu} \nabla_\gamma \sigma \nabla^\gamma \sigma - \nabla_\mu \sigma \nabla_\nu \sigma \right), \tag{5}$$

and \tilde{T} is the $g_{\mu\nu}$ trace of the energy-momentum tensor $\tilde{T}_{\mu\nu}$. Now, we introduce the scale transformations

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \tag{6}$$

$$\sqrt{-g} \rightarrow \Omega^4 \sqrt{-g}, \tag{7}$$

$$R \rightarrow \Omega^{-2} R + 6\Omega^{-3} \nabla_\mu \nabla_\nu \Omega g^{\mu\nu}, \tag{8}$$

where $\Omega = e^{-\sigma}$. The action (1) then becomes

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} [R\Omega^2 + 6\Omega\Box\Omega - 2\bar{\Lambda}\Omega^4 + \alpha g^{\mu\nu} \nabla_\mu \Omega \nabla_\nu \Omega] d^4x + S_m(g_{\mu\nu}), \tag{9}$$

where gravity couples non-minimally to the dilaton field and matter couples to the gravitational metric $g_{\mu\nu}$. The field equations are obtained by variation of (9) with respect to the fields $g_{\mu\nu}$ and Ω as

$$G_{\mu\nu} + \Omega^2 \bar{\Lambda} g_{\mu\nu} = \Omega^{-2} \bar{\kappa}^2 T_{\mu\nu} + \tau_{\mu\nu}(\Omega), \tag{10}$$

and

$$\Box\Omega - \frac{1}{(\alpha - 6)} (R - 4\bar{\Lambda}\Omega^2)\Omega = 0, \tag{11}$$

where

$$\tau_{\mu\nu}(\Omega) = (\alpha - 6)\Omega^{-2} \left[\frac{1}{2} g_{\mu\nu} \nabla_\lambda \Omega \nabla^\lambda \Omega - \nabla_\mu \Omega \nabla_\nu \Omega \right] - \Omega^{-2} [g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu] \Omega^2. \tag{12}$$

Manipulating the last term in (12) and taking the $g_{\mu\nu}$ trace of (10) gives

$$-R + 4\Omega^2 \bar{\Lambda} = \Omega^{-2} \bar{\kappa}^2 T + (\alpha - 6)\Omega^{-2} \nabla_\lambda \Omega \nabla^\lambda \Omega - 6\Omega^{-2} (\nabla_\lambda \Omega \nabla^\lambda \Omega + \Omega \Box\Omega). \tag{13}$$

It is easily seen that (11) and (13) are just consistent for $\alpha = 12$ which is a free parameter in the action. This value for α then leads the two equations to take the following forms

$$\Box\Omega - \frac{1}{6} (R - 4\bar{\Lambda}\Omega^2)\Omega = 0, \tag{14}$$

and

$$\Box\Omega - \frac{1}{6} (R - 4\bar{\Lambda}\Omega^2)\Omega = \Omega^{-2} \bar{\kappa}^2 T, \tag{15}$$

respectively. It turns out that (11) and (13) are consistent just for traceless energy-momentum tensors, namely $T = 0$. One may rewrite equation (10) as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu} + \tau_{\mu\nu}(\Omega), \tag{16}$$

where $\Lambda = \Omega^2 \bar{\Lambda}$ and $\kappa^2 = \Omega^{-2} \bar{\kappa}^2$. If we compare (11), for $\alpha = 12$, with the general form of the Ω field equation, namely $\Box\Omega + \frac{dV}{d\Omega} = 0$, we may infer the Higgs type potential for Ω as

$$V(\Omega) = -\frac{1}{12} (R - 2\bar{\Lambda}\Omega^2)\Omega^2. \tag{17}$$

For a given positive Ricci scalar, a negative $\bar{\Lambda}$ leads to vanishing minimum for the conformal factor, namely $\Omega_{min} = 0$. This case is a failure of conformal transformation with zero cosmological constant $\Omega^2 \bar{\Lambda}$, so is not physically viable. The nonvanishing minimum of this potential is obtained for a positive $\bar{\Lambda}$ as

$$\Omega_{min}^2 = \frac{R}{4\bar{\Lambda}} > 0. \tag{18}$$

Putting this value of Ω_{min} in (16) leads to the generalized Einstein field equation

$$G_{\mu\nu} = -\frac{R}{4}g_{\mu\nu} + \frac{4\bar{\Lambda}\bar{\kappa}^2}{R}T_{\mu\nu} + \frac{6}{R}\left[\frac{1}{2}g_{\mu\nu}(\nabla_\lambda\sqrt{R})(\nabla^\lambda\sqrt{R}) - (\nabla_\mu\sqrt{R})(\nabla_\nu\sqrt{R})\right] - \frac{1}{R}[g_{\mu\nu}\square - \nabla_\mu\nabla_\nu]R. \tag{19}$$

The important results are as follows:

- 1) When Ω or R is constant, the theory reduces to the standard GR with modified cosmological and gravitational constants, $\Lambda = \Omega^2 \bar{\Lambda}$, $\kappa^2 = \Omega^{-2} \bar{\kappa}^2$, respectively.
- 2) The field equations are consistent just for the matter fields for which $T = 0$. The model then reduces to GR with dynamical cosmological term $\frac{R}{4}g_{\mu\nu}$ and coupling term $\frac{4\bar{\Lambda}\bar{\kappa}^2}{R}$, and extra terms containing derivatives of the Ricci scalar.

3 Correspondence with Modified Gravity

The Einstein equations can be derived using the Palatini formalism, i.e., an independent variation with respect to the metric and an independent connection. The Riemann tensor and the Ricci tensor are also constructed with the independent connection and the metric is not needed to obtain the latter from the former. So, in order to make a difference with metric formalism, we shall use $\mathcal{R}_{\mu\nu}$ and \mathcal{R} instead of $R_{\mu\nu}$ and R , respectively. In this section we briefly review the $f(\mathcal{R})$ gravity in Palatini formalism [11]. The action in the Palatini formalism takes the form

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} f(\mathcal{R}) d^4x + S_m(g_{\mu\nu}). \tag{20}$$

Note that the matter action is assumed to depend only on the metric and the matter fields and not on the independent connection. This assumption is crucial for the derivation of Einstein’s equations from the action (20) and is the main feature of the Palatini formalism. Varying the action (20) independently with respect to the metric and the connection, respectively, and using the formula

$$\delta\mathcal{R}_{\mu\nu} = \bar{\nabla}_\lambda\delta\Gamma^\lambda_{\mu\nu} - \bar{\nabla}_\nu\delta\Gamma^\lambda_{\mu\lambda}, \tag{21}$$

yields

$$f'(\mathcal{R})\mathcal{R}_{(\mu\nu)} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \bar{\kappa}^2T_{\mu\nu}, \tag{22}$$

$$-\bar{\nabla}_\lambda(\sqrt{-g}f'(\mathcal{R})g^{\mu\nu}) + \bar{\nabla}_\sigma[\sqrt{-g}f'(\mathcal{R})g^{\sigma(\mu}]\delta_\lambda^{\nu)}) = 0, \tag{23}$$

where $\bar{\nabla}$ denotes the covariant derivative defined with the independent connection $\Gamma_{\mu\nu}^\lambda$ and $(\mu\nu)$ denotes symmetrization over the indices μ, ν . Taking the trace of (23) gives rise to

$$\bar{\nabla}_\sigma[\sqrt{-g}f'(\mathcal{R})g^{\sigma\mu}] = 0, \tag{24}$$

by which the field equation (23) becomes

$$\bar{\nabla}_\lambda(\sqrt{-g}f'(\mathcal{R})g^{\mu\nu}) = 0. \tag{25}$$

One may obtain some useful manipulations of the field equations. Taking the trace of (22) yields an algebraic equation in \mathcal{R}

$$f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = \bar{\kappa}^2 T. \tag{26}$$

For traceless energy-momentum tensors $T = 0$, the Ricci scalar \mathcal{R} will therefore be a constant root of the equation

$$f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = 0. \tag{27}$$

It is apparent from (26) that if $f(\mathcal{R}) \propto \mathcal{R}^2$ then only conformally invariant matter, for which $T = 0$ is identically satisfied, can be coupled to gravity [19]. One may define a metric conformal to $g_{\mu\nu}$ as

$$h_{\mu\nu} = f'(\mathcal{R})g_{\mu\nu}, \tag{28}$$

for which it is easily obtained that

$$\sqrt{-h}h^{\mu\nu} = \sqrt{-g}f'(\mathcal{R})g^{\mu\nu}. \tag{29}$$

Equation (25) is then the compatibility condition of the metric $h_{\mu\nu}$ with the connection $\Gamma_{\mu\nu}^\lambda$ and can be solved algebraically to give

$$\Gamma_{\mu\nu}^\lambda = h^{\lambda\sigma}(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}). \tag{30}$$

Under conformal transformations (28), the Ricci tensor and its contracted form with $g^{\mu\nu}$ transform, respectively, as

$$\begin{aligned} \mathcal{R}_{\mu\nu} &= R_{\mu\nu} + \frac{3}{2} \frac{1}{(f'(\mathcal{R}))^2} (\nabla_\mu f'(\mathcal{R}))(\nabla_\nu f'(\mathcal{R})) \\ &\quad - \frac{1}{(f'(\mathcal{R}))} \left(\nabla_\mu \nabla_\nu - \frac{1}{2} g_{\mu\nu} \square \right) f'(\mathcal{R}), \end{aligned} \tag{31}$$

$$\mathcal{R} = R + \frac{3}{2} \frac{1}{(f'(\mathcal{R}))^2} (\nabla_\mu f'(\mathcal{R}))(\nabla^\mu f'(\mathcal{R})) + \frac{3}{(f'(\mathcal{R}))} \square f'(\mathcal{R}). \tag{32}$$

Note the difference between \mathcal{R} and the Ricci scalar of $h_{\mu\nu}$ due to the fact that $g^{\mu\nu}$ is used here for the contraction of $\mathcal{R}_{\mu\nu}$. Substituting (31), (32) in the field equation (22), one obtains

$$\begin{aligned} G_{\mu\nu} &= \frac{\bar{\kappa}^2}{f'(\mathcal{R})} T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(\mathcal{R} - \frac{f(\mathcal{R})}{f'(\mathcal{R})} \right) + \frac{1}{f'(\mathcal{R})} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f'(\mathcal{R}) \\ &\quad - \frac{3}{2} \frac{1}{(f'(\mathcal{R}))^2} \left[(\nabla_\mu f'(\mathcal{R}))(\nabla_\nu f'(\mathcal{R})) - \frac{1}{2} g_{\mu\nu} (\nabla_\lambda f'(\mathcal{R}))(\nabla^\lambda f'(\mathcal{R})) \right]. \end{aligned} \tag{33}$$

In fact, since (26) relates \mathcal{R} algebraically with T , and that we have an explicit expression for $\Gamma_{\mu\nu}^\lambda$ in terms of \mathcal{R} and $h_{\mu\nu}$ (or $g_{\mu\nu}$) we can in principle eliminate the independent connection from the field equations and express them only in terms of the metric and the matter fields. Therefore, both sides of (33) depend only on the metric and the matter fields and the theory has been reduced to the form of GR with a modified source.

The results of this Palatini’s method for $f(\mathcal{R})$ gravity are as follows:

- 1) When $f(\mathcal{R}) = \mathcal{R}$, the theory reduces to the standard GR.
- 2) For the matter fields for which $T = 0$, the Ricci scalar \mathcal{R} and consequently $f(\mathcal{R})$ and $f'(\mathcal{R})$ are constants (due to (27)) and the theory reduces to GR with a cosmological constant and a modified coupling constant $\bar{\kappa}^2/f'$. If \mathcal{R}_0 is the value of \mathcal{R} when $T = 0$, then the value of the cosmological constant is

$$\frac{1}{2} \left(\mathcal{R}_0 - \frac{f(\mathcal{R}_0)}{f'(\mathcal{R}_0)} \right) = \frac{\mathcal{R}_0}{4}, \tag{34}$$

where use has been made of (27).

- 3) In the general case $T \neq 0$, the modified source on the right hand side of (33) includes derivatives of the stress-energy tensor which are implicit in the last two terms, since f' is in practice a function of T , namely $f' = f'(\mathcal{R})$ and $\mathcal{R} = \mathcal{R}(T)$.

It is easily shown that one may recover (19) through an equation similar to (33) in which $f(\mathcal{R})$ is replaced by $f(R)$ as

$$G_{\mu\nu} = \frac{\bar{\kappa}^2}{f'(R)} T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R - \frac{f(R)}{f'(R)} \right) + \frac{1}{f'(R)} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f'(R) - \frac{3}{2} \frac{1}{(f'(R))^2} \left[(\nabla_\mu f'(R)) (\nabla_\nu f'(R)) - \frac{1}{2} g_{\mu\nu} (\nabla_\lambda f'(R)) (\nabla^\lambda f'(R)) \right], \tag{35}$$

provided that

$$f(R) = \frac{1}{2} R^2. \tag{36}$$

Therefore, there is a *formal* correspondence between (33) and (35). Note that (35) is neither the same as (33) in Palatini formalism (according to (32), \mathcal{R} and R are not the same) nor the same as equation which is obtained in metric formalism as [11]

$$G_{\mu\nu} = \frac{\bar{\kappa}^2}{f'(R)} T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \frac{[f(R) - Rf'(R)]}{f'(R)} + \frac{1}{f'(R)} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f'(R). \tag{37}$$

4 Correspondence with Brans-Dicke Theory

The $f(\mathcal{R})$ gravity action coupled with matter is given by (20)

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} f(\mathcal{R}) d^4x + S_m(g_{\mu\nu}).$$

By introducing a new auxiliary field χ , the dynamically equivalent action is rewritten [11]

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} [f(\chi) + f'(\chi)(\mathcal{R} - \chi)] d^4x + S_m(g_{\mu\nu}). \tag{38}$$

Variation with respect to χ yields the equation

$$f''(\chi)(\mathcal{R} - \chi) = 0. \tag{39}$$

Redefining the field χ by $\phi = f'(\chi)$ and introducing $V(\phi) = \chi(\phi)\phi - f(\chi(\phi))$ the action takes the form

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g}[\phi\mathcal{R} - V(\phi)]d^4x + S_m(g_{\mu\nu}). \tag{40}$$

Now, we may use (32) which relates \mathcal{R} and R . To this end, we use $\phi = f'(\chi)$ subject to the constraint (39) so that we may replace $f'(\mathcal{R})$ by ϕ in (32) to obtain

$$\mathcal{R} = R + \frac{3}{2} \frac{1}{(\phi)^2} (\nabla_\mu \phi)(\nabla^\mu \phi) + \frac{3}{(\phi)} \square \phi. \tag{41}$$

Therefore, the action (40) modulo surface terms obtained by $\frac{3}{(\phi)} \square \phi$ can be rewritten as

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} \left(\phi R + \frac{3}{2\phi} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right) d^4x + S_m(g_{\mu\nu}). \tag{42}$$

This is the action of a Brans-Dicke theory with Brans-Dicke parameter $\omega = -\frac{3}{2}$. The generalized Einstein equation obtained by variation of the action (42) with respect to $g_{\mu\nu}$ is

$$\begin{aligned} G_{\mu\nu} = & \frac{\bar{\kappa}^2}{\phi} T_{\mu\nu} - \frac{3}{2\phi^2} \left(\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\lambda \phi \nabla^\lambda \phi \right) \\ & + \frac{1}{\phi} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \phi - \frac{V(\phi)}{2\phi} g_{\mu\nu}. \end{aligned} \tag{43}$$

It is easily shown that (19) corresponds to (43) provided that

$$\phi \equiv \Omega_{min}^2, \quad V(\phi) \equiv 12V(\Omega_{min}^2) = \frac{R^2}{8\Lambda}. \tag{44}$$

5 Conclusion

In this paper, we studied a model of scale transformation imposed on a gravitational action coupled with matter which is not scale invariant due to the presence of dimensional gravitational and cosmological constants. By choosing a dilaton field as a means of scale transformation we obtained a new action whose field equations are consistent for traceless matter and reveal dynamical cosmological term and gravitational coupling. We then examined the correspondence between the present model, the modified gravity theory, and Brans-Dicke theory.

First, we showed that the modified source terms in the obtained Einstein equation may be *formally* considered as equivalent to those of $f(\mathcal{R}) = \frac{1}{2}\mathcal{R}^2$ gravity in Palatini formalism, but with two differences: 1) Unlike the $f(\mathcal{R})$ gravity for a traceless matter $T = 0$ which reduces to GR with cosmological and modified coupling *constants*, the present modified Einstein equation subject to $T = 0$ has the advantage of non-vanishing modified terms together with *dynamical* cosmological and gravitational coupling terms. In other words, unlike the $f(\mathcal{R})$

gravity, the traceless matter in the present model does not reduce to GR and can still be considered as general as those matters with $T \neq 0$ which are studied in $f(\mathcal{R})$ gravity, 2) Unlike the $f(\mathcal{R})$ gravity in Palatini formalism where the modified source terms depend on the quantity \mathcal{R} which is not the Ricci scalar of $h_{\mu\nu}$, the modified source terms here depend on the Ricci scalar R constructed by the metric $g_{\mu\nu}$. Note that the above correspondence is obtained provided the Ω field is condensated to its vacuum state Ω_{min} . Overall, the generalized Einstein equation for traceless matter obtained here through scale transformation is neither an equation in $f(R)$ metric formalism nor the one in $f(\mathcal{R})$ Palatini formalism. Finally, we showed the present model corresponds to a Brans-Dicke theory with Brans-Dicke parameter $\omega = -\frac{3}{2}$.

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